Reteach

Using Inductive Reasoning to Make Conjectures

When you make a general rule or conclusion based on a pattern, you are using inductive reasoning. A conclusion based on a pattern is called a conjecture.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Conjecture</th>
<th>Next Two Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>−8, −3, 2, 7, . . .</td>
<td>Each term is 5 more than the previous term.</td>
<td>7 + 5 = 12, 12 + 5 = 17</td>
</tr>
</tbody>
</table>

Find the next item in each pattern.

1. \( \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \ldots \)

2. 100, 81, 64, 49, . . .

3. [Pattern Diagram]

4. [Pattern Diagram]

Complete each conjecture.

5. If the side length of a square is doubled, the perimeter of the square is ____________________________.

6. The number of nonoverlapping angles formed by \( n \) lines intersecting in a point is ____________________________.

Use the figure to complete the conjecture in Exercise 7.

7. The perimeter of a figure that has \( n \) of these triangles is ____________________________.
LESSON 2-1
Using Inductive Reasoning to Make Conjectures
continued

Since a conjecture is an educated guess, it may be true or false. It takes only one example, or counterexample, to prove that a conjecture is false.

Conjecture: For any integer \( n \), \( n \leq 4n \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n \leq 4n )</th>
<th>True or False?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( 3 \leq 4(3) ) ( 3 \leq 12 )</td>
<td>true</td>
</tr>
<tr>
<td>0</td>
<td>( 0 \leq 4(0) ) ( 0 \leq 0 )</td>
<td>true</td>
</tr>
<tr>
<td>(-2)</td>
<td>( -2 \leq 4(-2) ) ( -2 \leq -8 )</td>
<td>false</td>
</tr>
</tbody>
</table>

\( n = -2 \) is a counterexample, so the conjecture is false.

Show that each conjecture is false by finding a counterexample.

8. If three lines lie in the same plane, then they intersect in at least one point.

9. Points \( A \), \( G \), and \( N \) are collinear. If \( AG = 7 \) inches and \( GN = 5 \) inches, then \( AN = 12 \) inches.

10. For any real numbers \( x \) and \( y \), if \( x > y \), then \( x^2 > y^2 \).

11. The total number of angles in the figure is 3.

12. If two angles are acute, then the sum of their measures equals the measure of an obtuse angle.

Determine whether each conjecture is true. If not, write or draw a counterexample.

13. Points \( Q \) and \( R \) are collinear.

14. If \( J \) is between \( H \) and \( K \), then \( HJ = JK \).
Using Inductive Reasoning to Make Conjectures

Practice A

Find the next item in each pattern.

1. 2, 4, 6, 8, . . .
3. fat, winter, spring, . . .
   10 W summer

4. When several examples form a pattern and you assume the pattern will continue, you are applying inductive reasoning.
5. A statement you believe to be true based on inductive reasoning is called a conjecture.

For Exercises 6–8, complete each conjecture by looking for a pattern in the examples.
6. The sum of two odd numbers is even.
7. The number of sides of a polygon that has n vertices is n.
8. When a tree is cut horizontally, a series of rings is visible in the stump. Make a conjecture about the number of rings and the age of the tree based on the data in the table.

<table>
<thead>
<tr>
<th>Number of Rings</th>
<th>Age of Tree (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>22</td>
<td>60</td>
</tr>
</tbody>
</table>

9. Assume your conjecture in Exercise 8 is true. Find the number of rings in an 82-year-old oak tree.
10. A counterexample shows that a conjecture is false.

Show that each conjecture is false by finding a counterexample.
11. For any number n, 2n > n.
   Possible answers: zero, any negative number
12. Two rays having the same endpoint make an acute angle.
   (Sketch a counterexample.)
   Possible answer:

Using Inductive Reasoning to Make Conjectures

Practice B

Find the next item in each pattern.

1. 100, 81, 64, 49, . . .
   \[ . . . \]
2. 8, 13, . . .
   \[ . . . \]
   Arkansas
4. west, south, east, . . .
   north

Complete each conjecture.
5. The square of any negative number is positive.
6. The number of diagonals that can be drawn from one vertex in a convex polygon is \( n - 3 \).
7. For any integer \( n, n^2 > 0 \).
   Possible answers: zero, any negative number
8. Each angle in a right triangle has a different measure.
9. For many years in the United States, each bank printed its own currency. The variety of different bills led to widespread counterfeiting. By the time of the Civil War, a significant fraction of the currency in circulation was counterfeit. If one Civil War soldier had 48 bills, 16 of which were counterfeit, and another soldier had 39 bills, 13 of which were counterfeit, make a conjecture about what fraction of bills were counterfeit at the time of the Civil War.
   One-third of the bills were counterfeit.
   Make a conjecture about each pattern. Write the next two items.
10. 1, 2, 4, 8, 32, . . .
11. Each item, starting with the third, is the product of the two preceding items: 256, 8192.
    32

Using Inductive Reasoning to Make Conjectures

Practice C

Make a conjecture about each pattern. Write the next two items.

1. \(-1, -8, -27, -64, \ldots\)
   The pattern is the cubes of the negative integers: \(-125, -216, \ldots\).
2. \(1, 11, 21, 121, 11221, \ldots\)
   (Hint: Try reading the numbers aloud in different ways.)
   Each item describes the item before it (one, one, two ones, \ldots);
   
3. A, E, F, H, I, L, \ldots
   312211, 13112211.
   The pattern is the letters of the alphabet that are made only from straight segments: K, L.
4. \[ \square \bigtriangleup \triangle \cdots \square \bigtriangleup \triangle \cdots \]
   First rotate the figure 180°. Then reflect the figure across a vertical line. Repeat.
   Determine if each conjecture is true. If not, write or draw a counterexample.
5. Three points that determine a plane also determine a triangle.
   true
6. An image reflected across the x-axis cannot appear identical to its preimage.
   Sample answer:
   \[ \begin{array}{c}
   \text{original} \\
   \text{reflected}
   \end{array} \]
7. If \( a > b \) and \( b > c \), then \( a > b > c \).
   true
8. If \( n \) is an integer \( (n > 0) \), then \( \frac{1}{n} > \frac{1}{2} \).
   Possible answers: \( n = 1 \), \( n = -1 \)
9. Hank finds that a convex polygon with \( n \) sides has \( n - 3 \) diagonals from any one vertex. He notices that the diagonals from one vertex divide every polygon into triangles, and he knows that the sum of the angle measures in any triangle is 180°.
   Hank wants to develop a rule about the sum of the angle measures in a convex polygon with \( n \) sides. Find the rule. (Hint: Sketching some polygons may help.)
   Sum of angle measures = \( (180(n - 2)) \)
10. A regular polygon is a polygon in which all sides have the same length and all angles have the same measure. Use your answer to Exercise 9 to determine the measure of one angle in a regular heptagon, a regular nonagon, and a regular dodecagon. Round to the nearest tenth of a degree.
   \[ 128.6°, 140°, 150° \]

Using Inductive Reasoning to Make Conjectures

Retrace

Find the next item in each pattern.

1. \[ \frac{1}{2}, \frac{2}{3}, 1, \ldots \]
2. 2, \[ \square \bigtriangleup \triangle \cdots \square \bigtriangleup \triangle \cdots \]
3. \[ \begin{array}{c}
   \text{The dot skips over one vertex}
   \end{array} \]
   in a clockwise direction.
4. \[ \begin{array}{c}
   \text{The measure of each angle is half the measure of the previous angle.}
   \end{array} \]
   \[ \square \bigtriangleup \triangle \cdots \square \bigtriangleup \triangle \cdots \]

Complete each conjecture.
5. If the side length of a square is doubled, the perimeter of the square is ______ doubled.
6. The number of nonoverlapping angles formed by \( n \) lines intersecting in a point is \( 2n \).
7. The perimeter of a figure that has \( n \) of these triangles is ______ ______ ______ ______ ______ \( n + 2 \)
Using Inductive Reasoning to Make Conjectures continued

Since a conjecture is an educated guess, it may be true or false. It takes only one example, or counterexample, to prove that a conjecture is false.

Conjecture: For any integer n, n ≤ 4n.

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<th>n</th>
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<tr>
<td>3</td>
<td>3 ≤ 4(3)</td>
<td>True</td>
</tr>
<tr>
<td>0</td>
<td>0 ≤ 4(0)</td>
<td>True</td>
</tr>
<tr>
<td>-2</td>
<td>-2 ≤ 4(-2)</td>
<td>False</td>
</tr>
</tbody>
</table>

n = -2 is a counterexample, so the conjecture is false.

Show that each conjecture is false by finding a counterexample.

8. If three lines lie in the same plane, then they intersect in at least one point.
   Possible answer: If the lines are parallel, then they do not intersect.

9. Points A, G, and N are collinear. If AG = 7 inches and GN = 5 inches, then AN = 12 inches.
   Possible answer: If point N is between points A and G, then AN = 2 inches.

10. For any real numbers x and y, if x > y, then x² > y².
    Sample answer: If x = 0 and y = -1, then x² < y².

11. The total number of angles in the figure is 3.
    Sample answer: ∠ABD, ∠DBE, ∠EBC, ∠CBE, ∠ABC.

12. If two angles are acute, then the sum of their measures equals the measure of an obtuse angle.
    Sample answer: ∠m/2 = 20°

Determine whether each conjecture is true. If not, write or draw a counterexample.

13. Points O and R are collinear.
    14. If J is between H and K, then HJ = JK.

   Sample answer: J is not between H and K.

Using Inductive Reasoning to Make Conjectures

The table shows the lengths of five green iguanas after birth and then after 1 year.

<table>
<thead>
<tr>
<th>Iguana</th>
<th>Length after Hatching</th>
<th>Length after 1 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>37</td>
</tr>
</tbody>
</table>

1. Estimate the length of a green iguana after 1 year if it was 8 inches long when it hatched.
   Sample answer: 12 in.

2. Make a conjecture about the average growth of a green iguana during the first year.
   Sample answer: The average growth of a green iguana during the first year is about 2 ft.

The times for the first eight matches of the Santa Barbara Open women's volleyball tournament are shown. Show each conjecture is false by finding a counterexample.

<table>
<thead>
<tr>
<th>Match</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>0.31</td>
<td>0.56</td>
<td>0.51</td>
<td>0.18</td>
<td>0.50</td>
<td>0.34</td>
<td>1.00</td>
<td>0.36</td>
</tr>
</tbody>
</table>

3. Every one of the first eight matches lasted less than 1 hour.
   Match 7 lasted 1 hour 3 minutes. Match 4 was 18 minutes long.

Choose the best answer.

5. The table shows the number of cells present during three phases of mitosis. If a sample contained 80 cells during interphase, which is the best prediction for the number of cells present during prophase?
   A 18 cells  C 40 cells
   B 24 cells  D 80 cells

6. About 75% of the students at Jackson High School volunteer to clean up a half-mile stretch of road every year. If there are 408 students in the school this year, about how many are expected to volunteer for the clean-up?
   F 102 students  H 306 students
   G 204 students  J 333 students

Using Inductive Reasoning to Make Conjectures

The steps to making a decision based on inductive reasoning are presented below.

1. Observe the pattern of the points on the grid.
2. Consider the two conjectures given to explain the pattern seen on the grid.
3. Evaluate each of the conjectures and plot the next two points on the graph according to each conjecture. Which conjecture is correct?

For Conjecture A, students should plot (5, 3) and (6, 2).
For Conjecture B, students should plot (5, 5) and (6, 6).
Student should conclude that Conjecture B is correct.